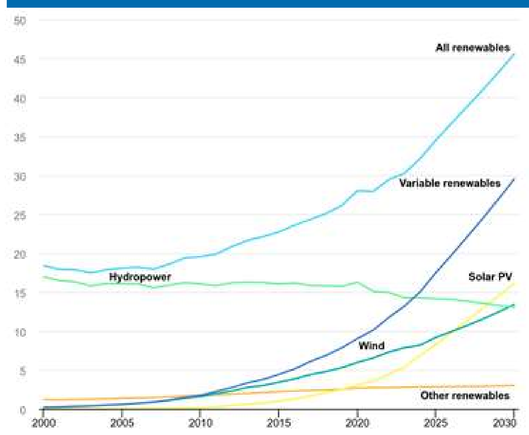
Analytical Transient-Stability Model and Stability Enhancing Compensation for GFM Inverters

Hyeong-Jun*, Saehwan Lim, So-seul Jeong, and Jin-Hong Jeon
Korea Electrotechnology Research Institute (KERI)

Introduction



Renewables provided record 30% of global electricity in 2023, Ember says

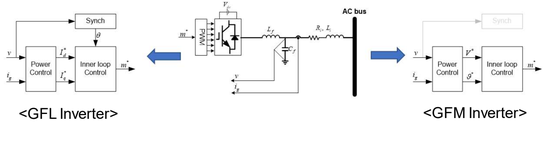
Climate change indicators reached record levels in 2023: WMO

PRESS RELEASE

19 March 2024

The state of the climate in 2023 gave ominous new significance to the phrase "off the charts."

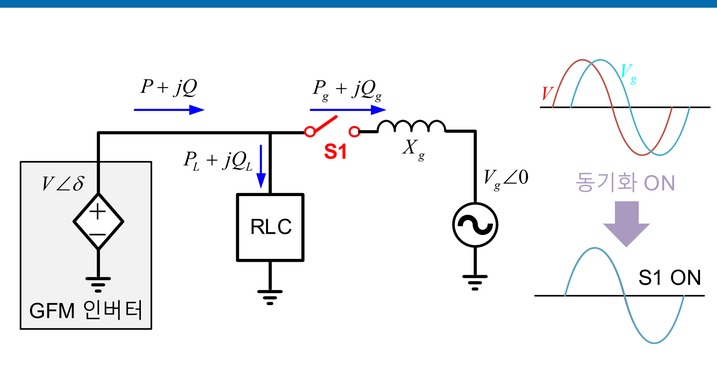
Key messages



<GFL Inverter> <GFM Inverter>

- With the rising share of IBRs, weak-grid(low SCR) conditions make the stability of GFM inverter a primary concern.
- The LPF in the P-loop and digital/compute delays of droop-based GFM reduce phase margin(PM), causing ringing or divergence under steps and line changes.
- We establish a mathematical $\delta - P$ model including R/X and enabled virtual impedance, a lead compensator effectively cancels the lag

Fundamental of Grid-Forming Inverters



<Re-synchronization and Re-connection of GFM inverters>

$$P(V,\delta,X_g) = P_L + P_g = \frac{3}{2}\cdot\frac{V^2}{R} + \frac{3}{2}\cdot\frac{VV_g}{X_g}\cdot\sin\delta \qquad V = V_0 + K_q(Q_{\text{ref}} - Q)$$

$$Q(V,\delta,X_g) = Q_L + Q_g = \frac{3}{2}V^2\left(\frac{1}{\omega_1 L} - \omega_1 C\right) + \frac{3}{2}\cdot\frac{V^2 - VV_g\cos\delta}{X_g}$$

$$V(\delta,X_g) = \frac{\left(1.5D_qE_g\cos\delta - X_g\right) + \sqrt{\left(X_g - 1.5D_qE_g\cos\delta\right)^2 + 6D_qX_g\left(1+\frac{X_gQ_fm}{R}\right)\left(D_qQ_{ref}+V_0\right)}}{3D_q\left(1+\frac{X_gQ_fm}{R}\right)}$$

$$P(\delta,V) = \frac{3}{2}\cdot\frac{V(\delta,X_g)^2}{R} + \frac{3}{2}\cdot\frac{V(\delta,X_g)V_g}{X_g}\cdot\sin\delta$$

<Power flow analysis vs. phase angle and voltage magnitude>

$$P = \frac{V_1^2}{Z}\cos\theta - \frac{V_1V_2}{Z}\cos\left(\theta+\delta_1-\delta_2\right)$$

$$Q = \frac{V_1^2}{Z}\sin\theta - \frac{V_1V_2}{Z}\sin\left(\theta+\delta_1-\delta_2\right)$$

Type of Line	R (Ω/km)	X (Ω/km)	R/X (p.u.)
Low Voltage Line	0.642	0.083	7.7
Medium Voltage Line	0.161	0.190	0.85
High Voltage Line	0.06	0.191	0.31

<Line impedance by AC Voltage level>

Grid-Forming Inverters Transitioning from Islanded to Grid-connected Mode

Active power loop during resynchronization process

$$\theta(s) = \frac{1}{s}\left[K_p\frac{\omega_c}{s+\omega_c}\left(P_{\text{ref}} - P\right) + \Delta\omega_{sync}\right] + \frac{\omega_1}{s}$$

The dynamic behavior of δ

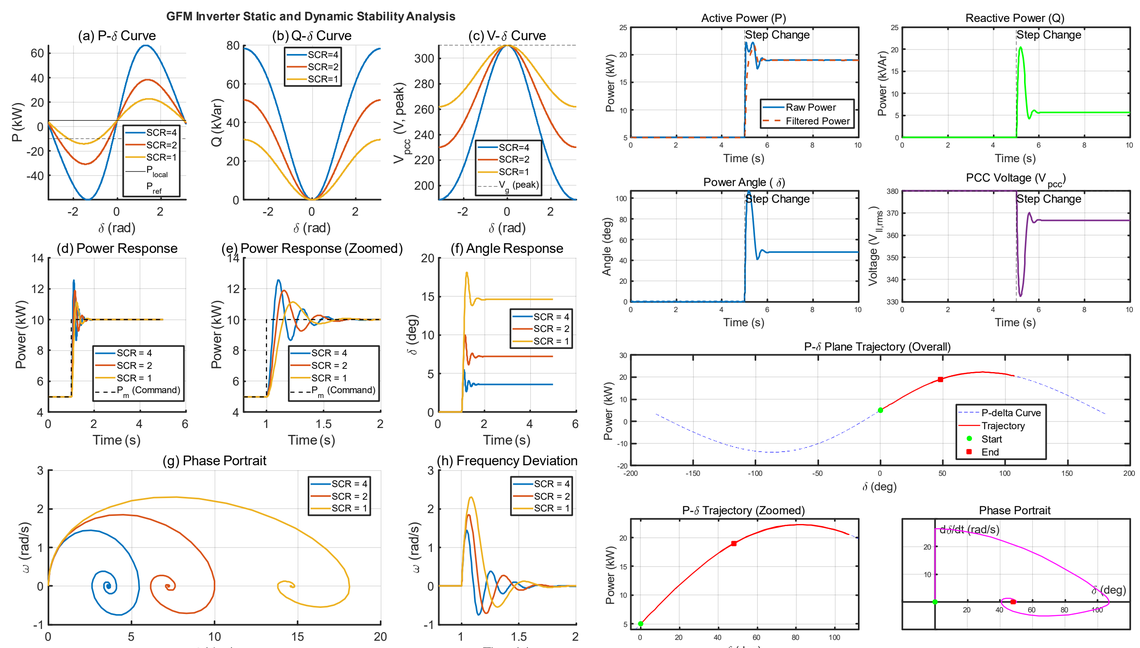
$$\ddot{\delta} = -\omega_c\dot{\delta} + \omega_c K_p\left(P_{\text{ref}} - P(\delta,\dot{\delta})\right).$$

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -\omega_c x_2 + \omega_c K_p\left(P_{\text{ref}} - P(x_1,x_2)\right).$$

$$\begin{cases} \dfrac{d\delta}{dt} = m_p\left(P_{ref}(t) - P_{filt}\right) \\ \dfrac{dP_{filt}}{dt} = \omega_c\left(P_{inv}(\delta,V) - P_{filt}\right) \end{cases}$$

The dynamic behavior of δ with the consideration of Q-V droop (including R/X ratio, virtual impedance)

$$\frac{1}{\omega_c K_p}\ddot{\delta}\dot{\delta} = -\frac{1}{K_p}\dot{\delta}^2 + \left(P_{\text{ref}} - P(\delta,\dot{\delta})\right)\dot{\delta}.$$



Lead Compensation for Improving Phase Margin

- **(Problems)** The LPF in the P-loop of droop-based GFM and the loop/compute delays reduce phase margin(PM)
- With varying SCR and R/X ratio, the reduced PM leads to oscillations and even divergence
- **(Approach)** Compensate the phase lag caused by LPF and delays using a lead compensator

For the power error $e_P = P^* - P$, define a 1st-order lead state:

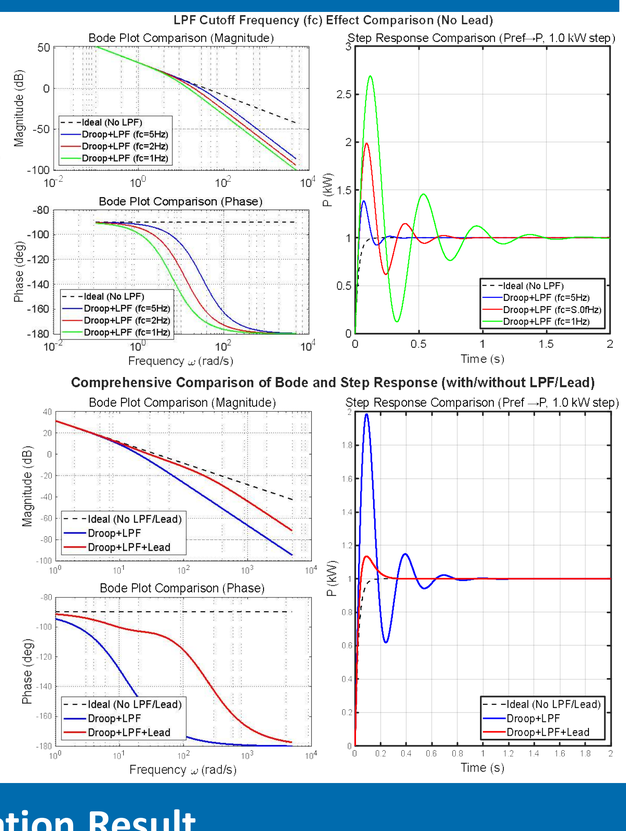
$$x_{lead} = \frac{1}{T}\left(e_P - x_{lead}\right), \qquad P_{comp} = \alpha e_P + \left(1-\alpha\right)x_{lead}, \qquad P_{final} = k_{att}P_{comp}$$

Transfer form (canonical 1-lead) & Lead coefficient:

$$G_{lead}(s) = k_{att}\frac{1+\alpha Ts}{1+Ts}, \ (\alpha>1) \ \alpha = \frac{1+\sin\phi_{req}}{1-\sin\phi_{req}}, \ T = \frac{1}{\omega_{target}\sqrt{\alpha}}, \ k_{att} = \frac{1}{\sqrt{\alpha}}$$

Open-loop Transfer function:

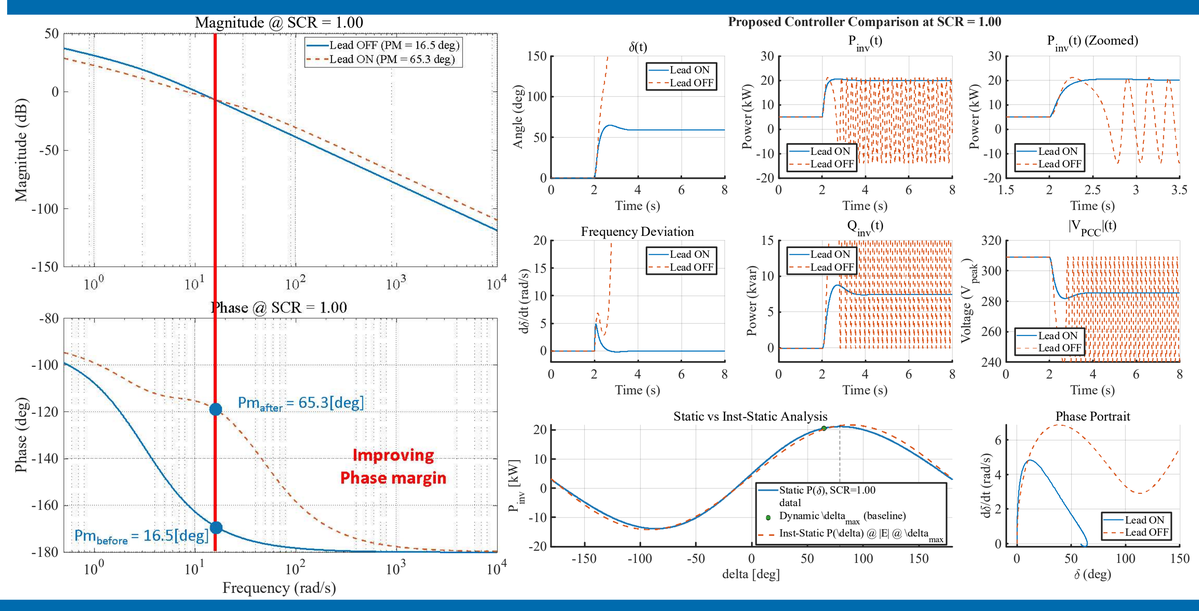
$$G_{ol}(s) = m_p K_{sync}\cdot\frac{1}{s}\cdot G_{LPF}(s)\cdot G_{lead}(s), \qquad G_{LPF}(s) = \frac{1}{1+\tau_P s}$$



Conclusion

- The LPF in the droop P-loop reduces phase margin (PM), causing instability/divergence under weak grids.
- This paper proposed a lead compensator with a clear design workflow and develop a closed-form δ–P model including R/X.
- The model matches EMT, enabling predictive design of PM, ROCOF, settling time, and the δ–P nose.
- With lead, PM increases (e.g., 16.5°→65.3°), ROCOF peak drops, settling shortens, and the
- δ trajectory stays below the nose and converges.

Simulation Result



- Setup: LPF = 1Hz, SCR = 1, Pref: 5kW → 20kW
- Baseline (no lead): LPF-induced phase lag yields small/negative PM → under steps/line changes, oscillations grow and the δ trajectory crosses the P-δ nose, leading to loss of synchronism.
- Proposed (with lead): PM restored (65) near wc → lower ROCOF peak, shorter settling, and suppressed high-frequency ringing. The delta trajectory stays below the nose and converges.
- Model-simulation agreement: Nose point/slope of the P- δ curve and time-domain metrics(ROCOF, settling) match EMT
- Mechanism: PM → closed-loop damping → operation within the valid $\partial P/\partial\delta$ range → convergence instead of divergence

KERI
한국전기연구원
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