

1. Problem Statement

- To analyze a system accurately, all of its parameters, initial conditions, and inputs are needed; however, finding their precise value is not possible in practice.
- Uncertainty of a system cannot be considered in conventional methods for system analysis.
- It is not possible to use simulation-based techniques to study the behavior of a system in all the possible scenarios.
- Reachability analysis can be used to analyze a system and consider its specific continuous set of state matrix, inputs and initial conditions. Reachability analysis finds the reachable sets for the system states with unknown-but-bounded state matrix, inputs and initial conditions.

2. Objective

- Analyze a system in all possible conditions and scenarios.
- Design a controller for a system with unknown-but-bounded state matrix using Reachability analysis.

3. Reachability Analysis

- For reachability analysis of a continuous linear system, its dynamic model in needed.

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t), \text{ with } x_0 \in \mathcal{X}_0 \subset R^n, u(t) \in \mathcal{U} \subset R^m$$

- A method based on zonotopes is used to compute reachable sets.
- The reachable set that includes all possible trajectories of the states through the time can be computed by solving the dynamic model.
- Using reachability analysis helps solve the system with unknown-but-bounded state matrix, initial conditions, and inputs.

4. Study System

- Grid-connected voltage-sourced converter (VSC)

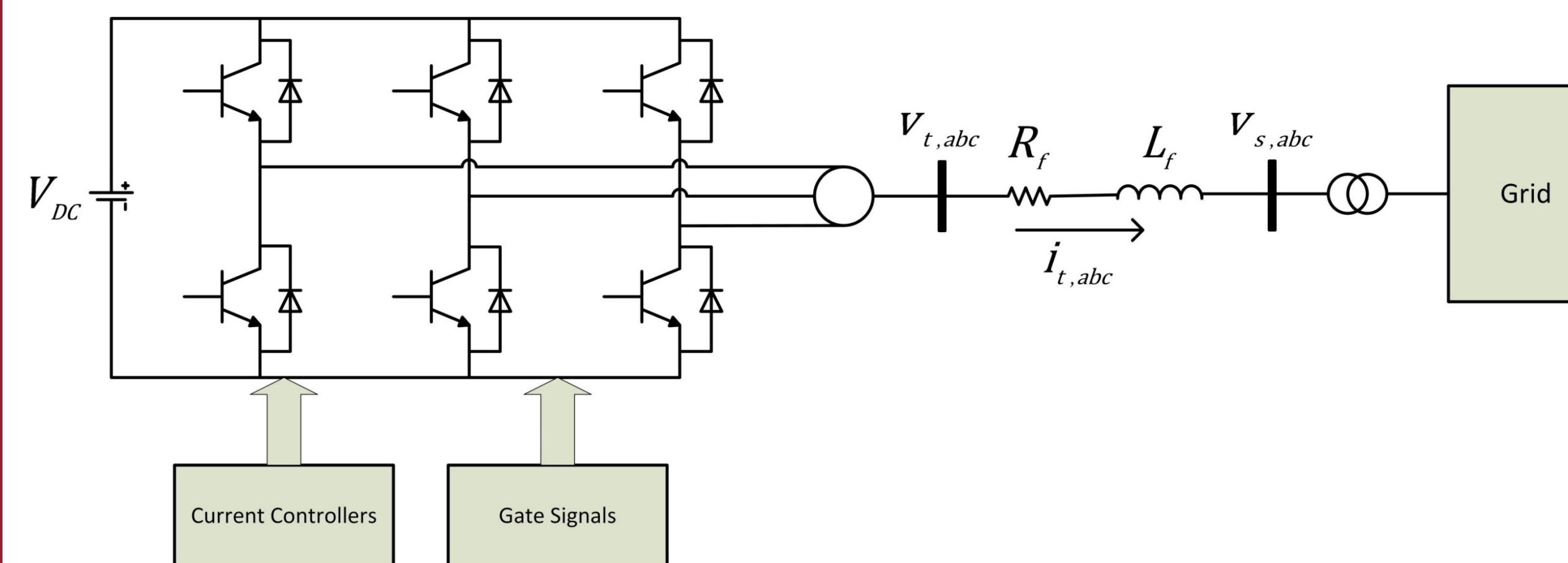


Fig. 1. Schematic diagram of the three phase VSC

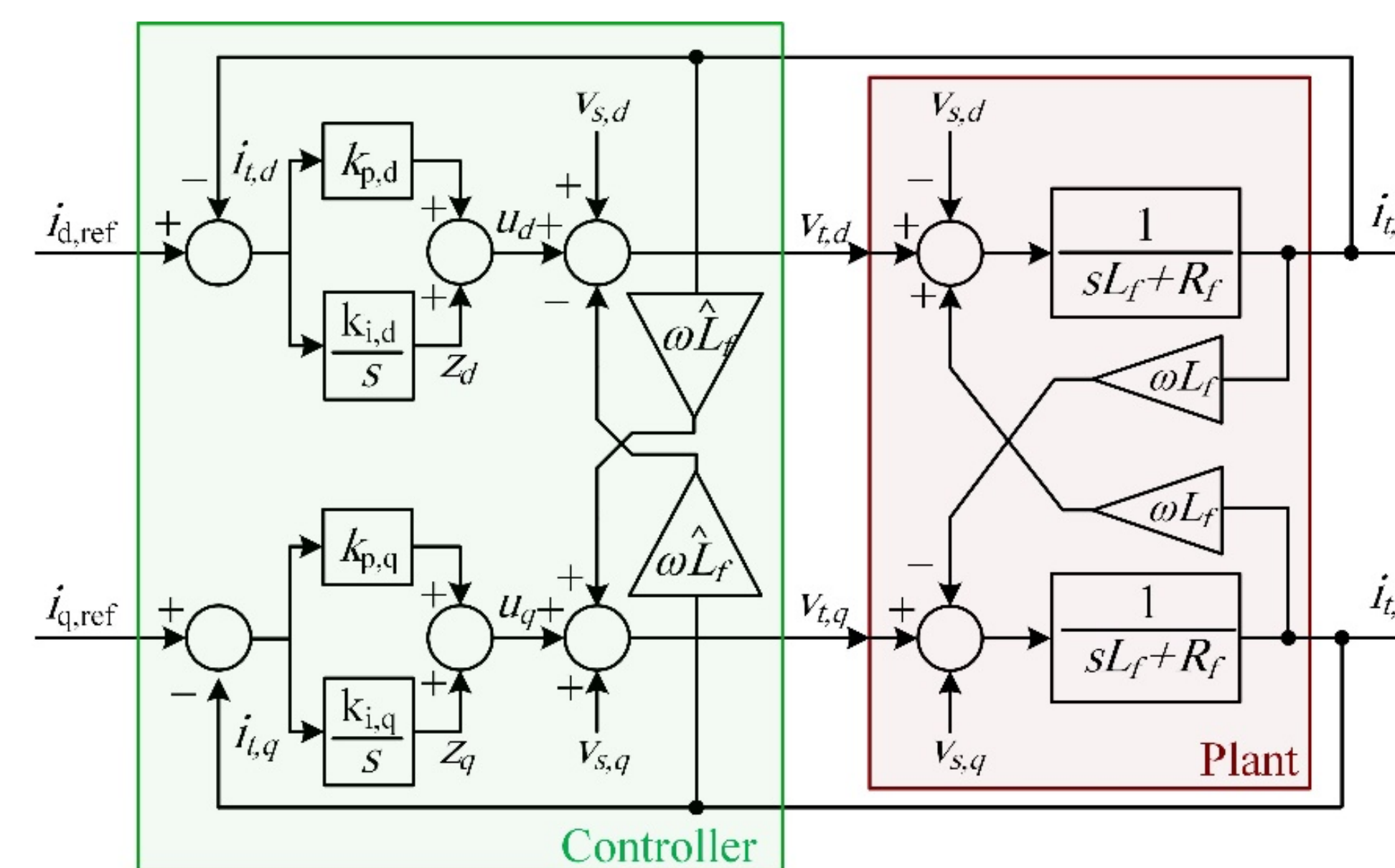


Fig. 2. Block diagram of the current controller of the VSC

- Dynamic model of a VSC in dq-frame

$$\frac{d}{dt} \begin{bmatrix} i_{t,d} \\ i_{t,q} \\ z_d \\ z_q \end{bmatrix} = \begin{bmatrix} -\frac{R_f + k_{p,d}}{L_f} & \frac{L_f - \hat{L}_f}{L_f} \omega & \frac{1}{L_f} & 0 \\ -\frac{L_f - \hat{L}_f}{L_f} \omega & -\frac{R_f + k_{p,q}}{L_f} & 0 & \frac{1}{L_f} \\ -k_{i,d} & 0 & 0 & 0 \\ 0 & -k_{i,q} & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{t,d} \\ i_{t,q} \\ z_d \\ z_q \end{bmatrix} + \begin{bmatrix} \frac{k_{p,d}}{L_f} & 0 \\ 0 & \frac{k_{p,q}}{L_f} \\ k_{i,d} & 0 \\ 0 & k_{i,q} \end{bmatrix} \begin{bmatrix} i_{d,ref} \\ i_{q,ref} \end{bmatrix}$$

- It is assumed that the measured inductance of the filter is not accurate and may have $\pm 20\%$ error.
- Inputs: $i_{q,ref} \in [-I_{nom}, I_{nom}]$ and $i_{d,ref} \in [0, I_{nom}]$.
- Initial PI controller is designed based on Bode plot method.
- Reachability analysis is used to design the PI controller by trial and error. The PI controller with the lowest limits in reachability analysis results is considered as the improved controller. Current limits decrease 4.3% by this method.

5. Reachability, Simulation, and Experimental Results

- Experimental setup

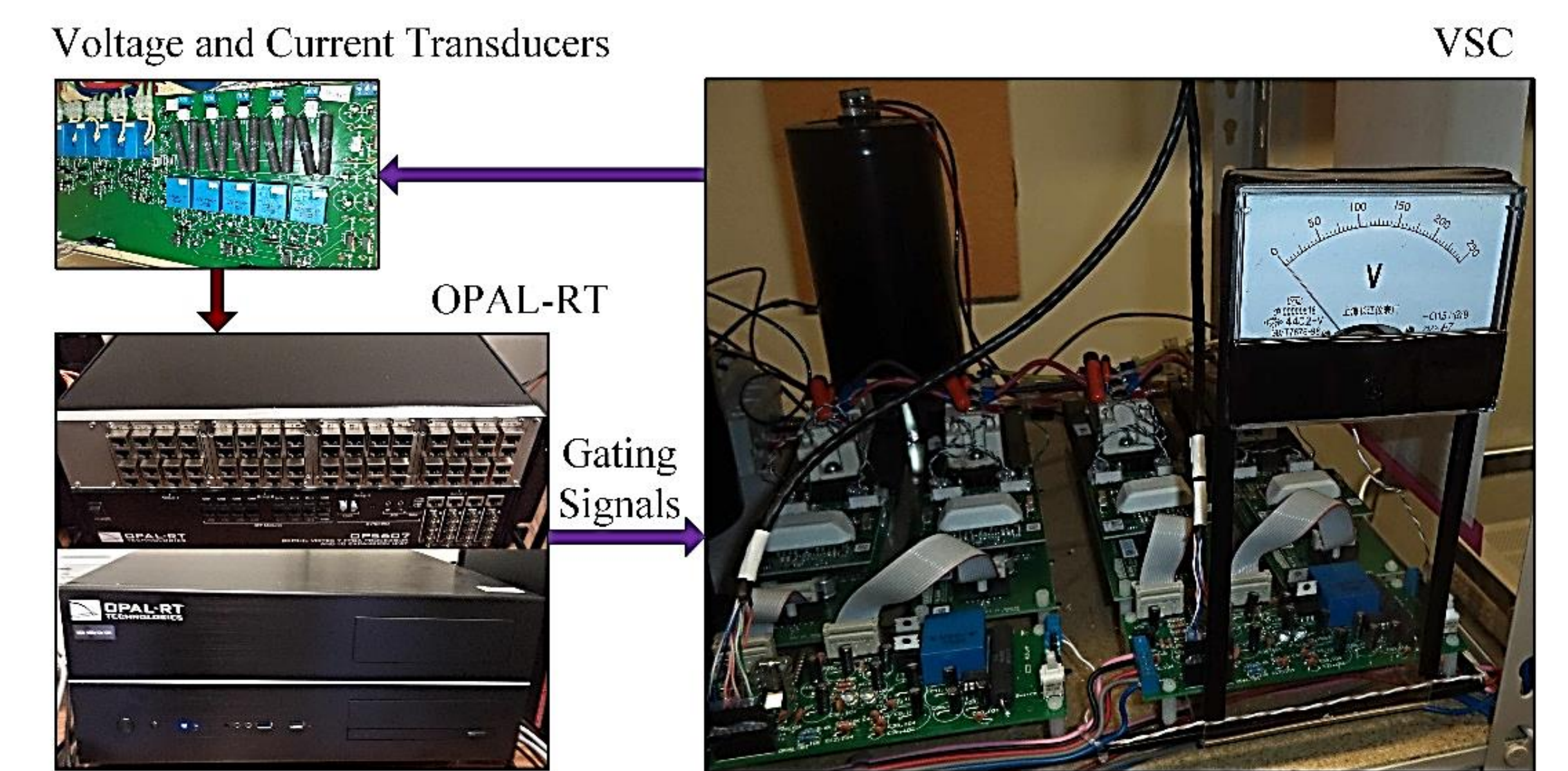


Fig. 3. Experimental setup of the grid-connected three phase VSC

- Reachability results vs simulation results

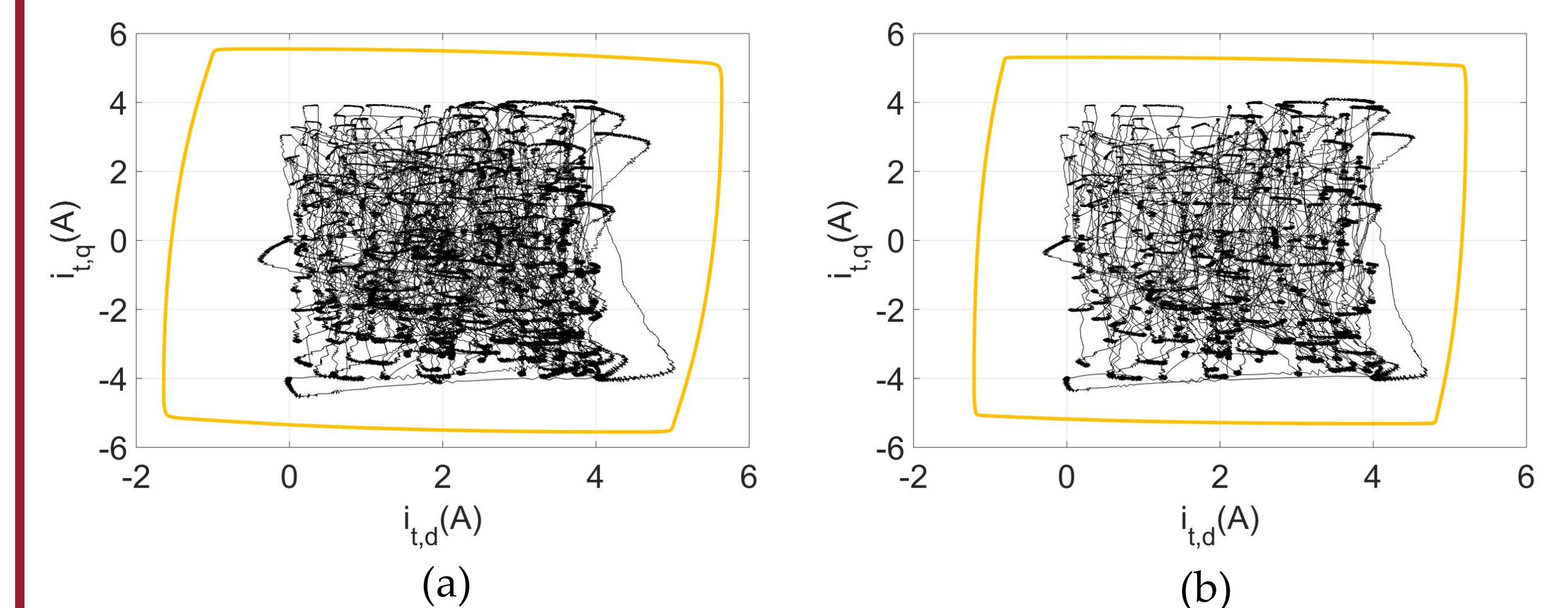


Fig. 4. Reachability analysis vs. simulation results for VSC terminal current with: (a) Initial PI controller based on Bode plot method; (b) designed PI controller by reachability analysis.

- Reachability results vs experimental results

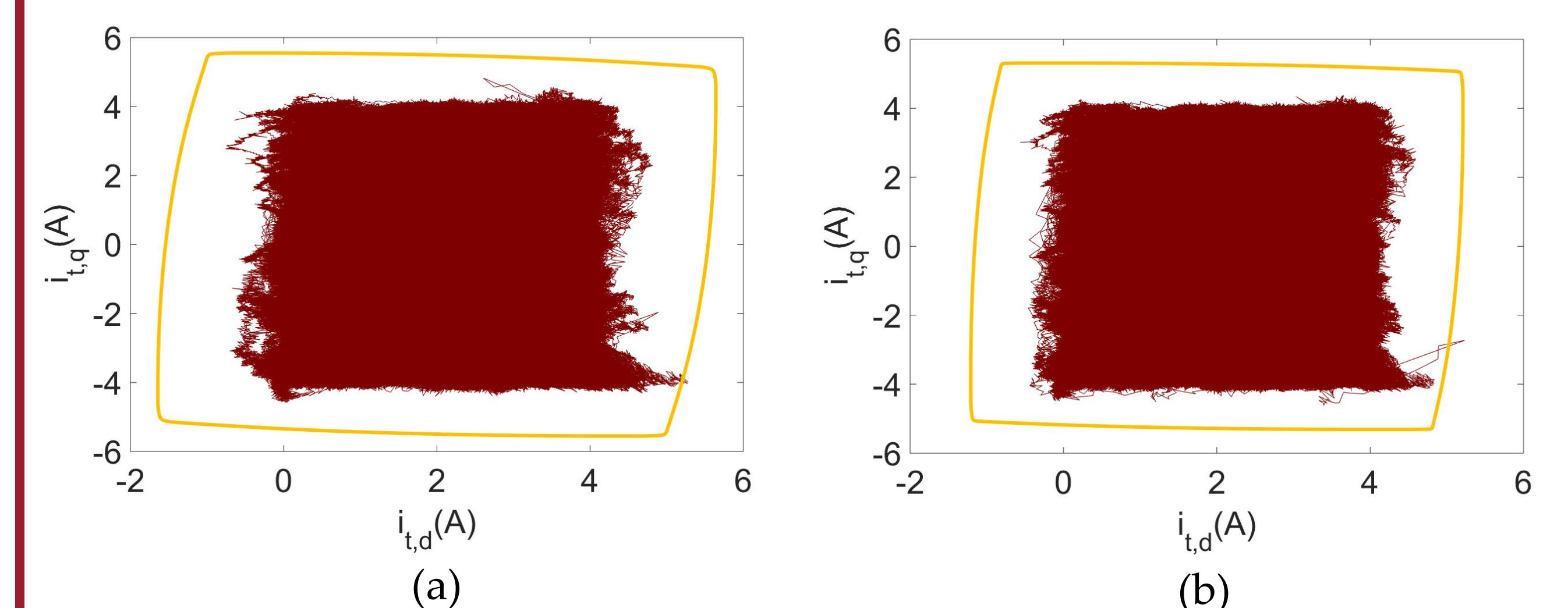


Fig. 5. Reachability analysis vs experimental results for VSC terminal current with: (a) Initial PI controller based on Bode plot; (b) designed PI controller by reachability analysis.